Averaging Methods for Heat Storage Control and Design Optimization

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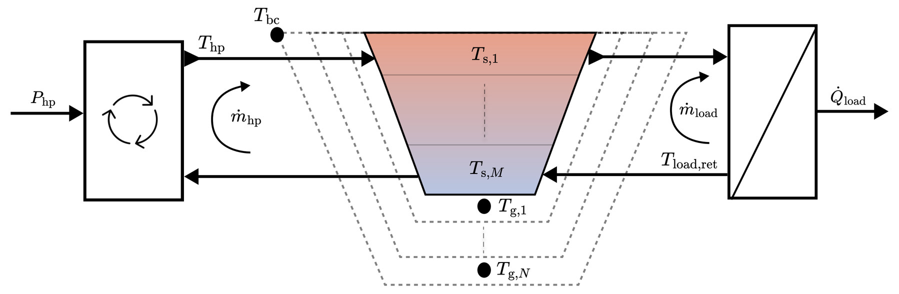
Introduction

- Seasonal underground heat storage can be used to store summer's surplus solar energy
- We optimize its design and hourly control strategy
- The simulation over a year is very costly and slow

Optimization Problem

$$\begin{array}{ll} \underset{\substack{\text{(control strategy)} \\ \text{(design parameters)}}}{\text{minimize}} & \text{(Total Cost)} \\ \text{subject to} & \text{(year-periodic state simulation),} \\ & \text{(operational constraints),} \\ & \text{(satisfy demands)} \end{array}$$

Heat Storage Dynamics



State Vector with Storage and Ground Temperatures

Ambient Temperature (Data)

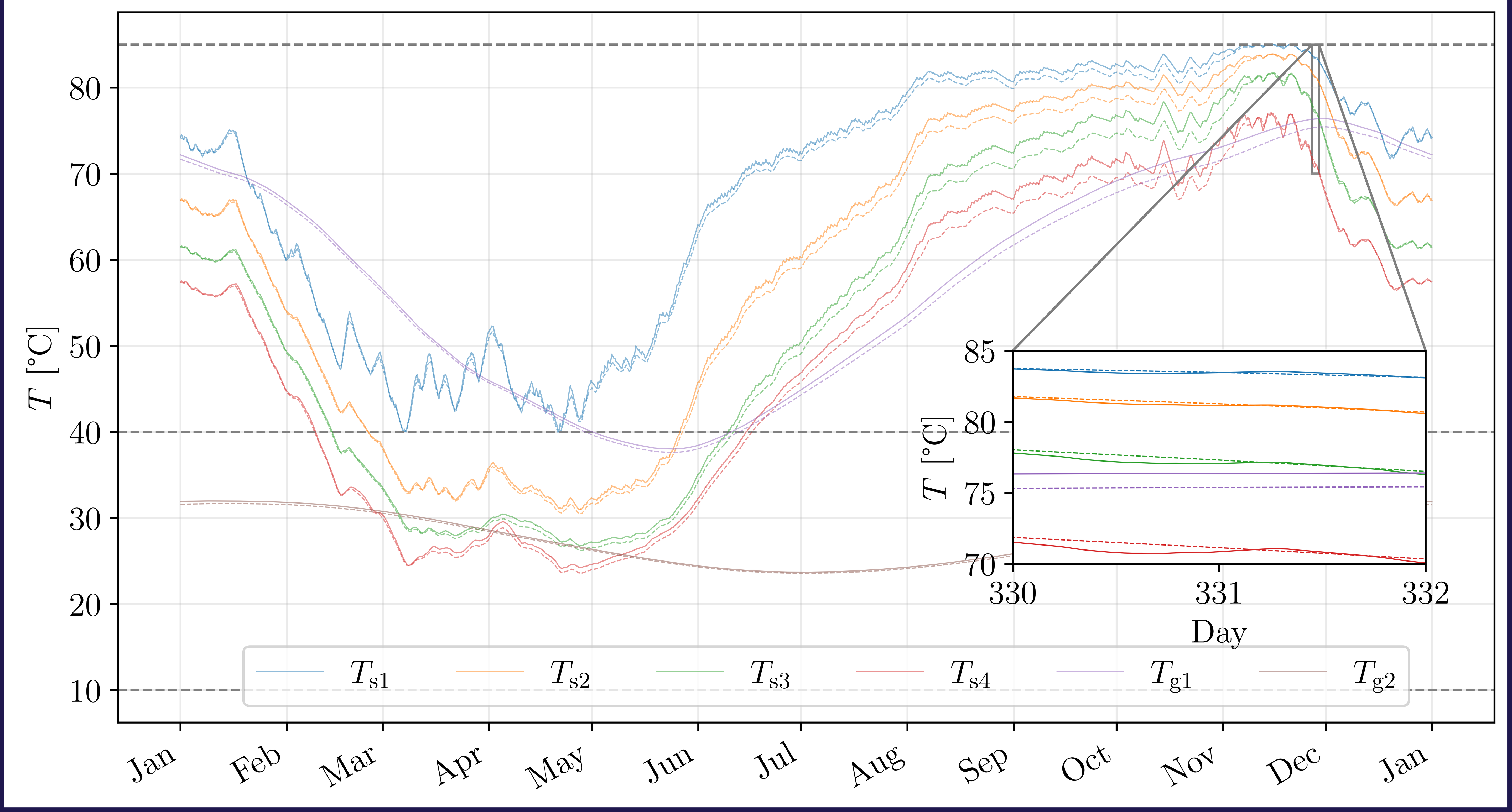
$$\dot{x^{\mathrm{s}}} = f^{\mathrm{s}}\left(x^{\mathrm{s}}, \dot{Q}_{\mathrm{hp}}(t), \dot{Q}_{\mathrm{load}}(t), T_{\mathrm{amb}}(t)\right)$$

Heat Inflow (Control)

Heat Outflow (Data)

State dynamics are **affine** in the 3 latter inputs!

For a year-long optimization of heat-storage operation, **averaged dynamics** are sufficiently accurate.



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Averaging Theorem

Lemma 1 (Averaging Theorem). *Consider the solution $x(t)$ of the initial value problem*

$$\dot{x}(t) = \epsilon f(x(t), t), \qquad x(0) = a \tag{B.1}$$

where the dynamics $f^1 : \mathbb{R}^{n_x} \times \mathbb{R}$ are Lipschitz continuous in (x, t) and T-periodic in t, and $0 < \epsilon \ll 1$. Then, the solution $y(t)$ of the averaged dynamics

$$\dot{y}(t) = \epsilon \bar{f}(y(t)) = \epsilon \frac{1}{T} \int_0^T f(y(t), \tau)\, d\tau \tag{B.2}$$

with initial value $y(0) = a$ approximates the solution of the original dynamics as $y(t) = x(t) + \mathcal{O}(\epsilon)$ on the timescale $1/\epsilon$.

Daily Average Dynamics

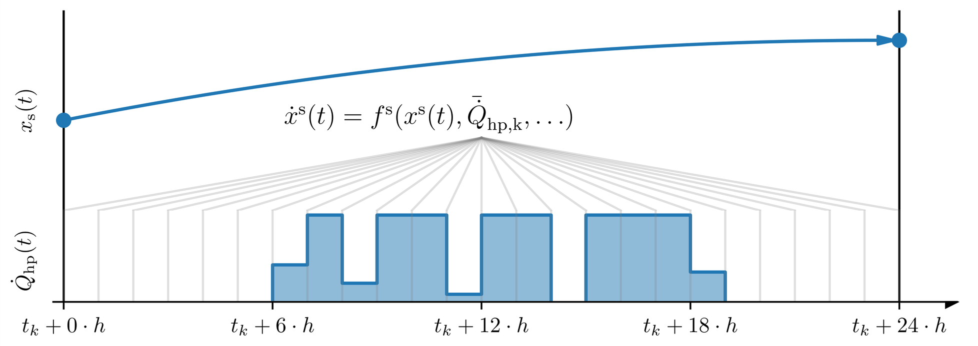
$$\begin{aligned} \bar{f}^{\mathrm{s}}_k(x^{\mathrm{s}}(t)) &= \frac{1}{T} \int_{t_k}^{t_k+T} f^{\mathrm{s}}\left(x^{\mathrm{s}}(t), \dot{Q}_{\mathrm{hp}}(\tau), \dot{Q}_{\mathrm{load}}(\tau), T_{\mathrm{amb}}(\tau)\right) d\tau \\ &= f^{\mathrm{s}}\left(x^{\mathrm{s}}(t), \bar{\dot{Q}}_{\mathrm{hp},k}, \bar{\dot{Q}}_{\mathrm{load},k}, \bar{T}_{\mathrm{amb},k}\right) \end{aligned}$$

Due to Affine Dynamics

Day-Averages of Controls or Data e.g.

$$\bar{\dot{Q}}_{\mathrm{load},k} := \frac{1}{T} \int_{t_k}^{t_k+T} \dot{Q}_{\mathrm{load}}(t)\, dt = \frac{1}{K} \sum_{i=k}^{i=k+K-1} \dot{Q}_{\mathrm{load},i}$$

We can now simulate with large steps in order to reduce the size of the nonlinear program.



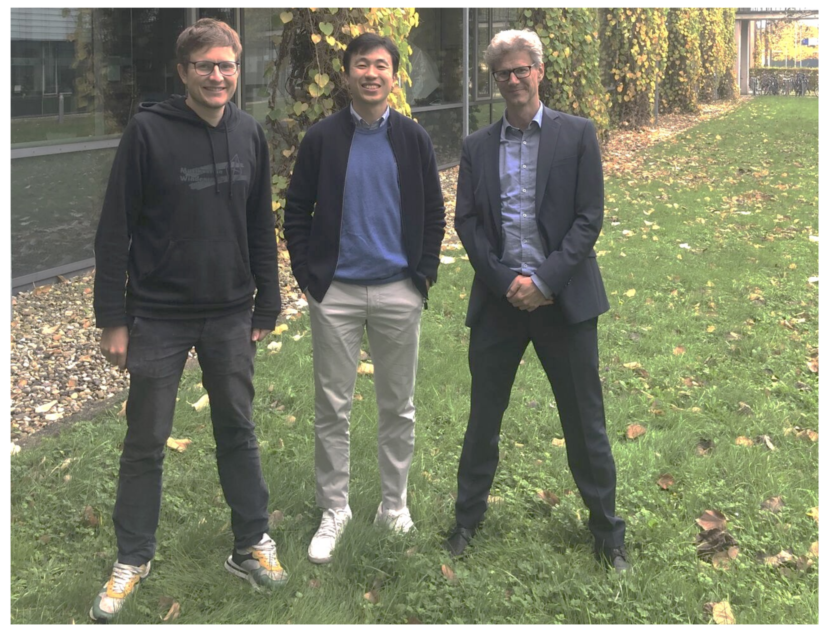
Speedups

	Original Problem	Averaging Method	Unit
Total Variables	105 130	54 762	
Time per Iteration	0.581	0.122	s

with mean abs. error of only 0.2%.

Implementation on Github

github.com/JakobHarz/NOSTES



Left to Right: Jakob Harzer, Wonsun Song, Moritz Diehl